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SELF-SUSTAINED OSCILLATIONS AND BIFURCATIONS OF TRANSONIC FLOW PAST SIMPLE AIRFOILS

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A turbulent flow past two symmetric airfoils, whose bow and aft portions are circular arcs, whereas midparts are flat, is studies numerically. The amplitude of lift coefficient oscillations versus the free-stream Mach number M_∞ *is analyzed at zero angle of attack. Ranges of* M_∞ *in which there exist flow bifurcations are determined.*

Key words: *airfoil, local supersonic zones, shock waves, buffet, bifurcations.*

Introduction. A characteristic feature of supercritical airfoils designed for transonic flight is a small local curvature of their upper surface [1–3]. At the same time, physical phenomena associated with the flow past such airfoils have not been well understood. Numerical simulations based on the system of Euler equations demonstrated bifurcations of a steady inviscid flow at certain angles of attack and free-stream Mach numbers [4–7]. The bifurcations and non-uniqueness of the transonic flow are caused by an instable interaction of two local supersonic zones residing on the same surface of the airfoil [8, 9].

A turbulent flow past an airfoil with a blunt nose and a cusped trailing edge, described by the formula

$$y(x) = \pm 0.06\sqrt{1-(2x-1)^4}\,(1-x^{16})^2 \qquad (0 \leqslant x \leqslant 1), \tag{1}$$

was studied in [10]. Computations confirmed the existence of bifurcations obtained previously for an inviscid flow. In addition, the numerical simulations revealed excitation of self-sustained flow oscillations due to instability of shock-induced boundary layer separation in the aft region of the airfoil.

In this paper, we study a turbulent flow past simple airfoils whose midparts are flat. Two cases with the airfoil thickness of 0.08 and 0.07, respectively, are analyzed.

1. Formulation of the Problem and Numerical Method. We consider a two-dimensional flow past a symmetric airfoil of thickness h, whose midpart is flat

$$y(x) = \pm h/2, \qquad a \leqslant x \leqslant 1-a, \tag{2a}$$

whereas the bow and aft portions are constituted by circular arcs of radius R that join smoothly the straight segments at $x = a$ and $x = 1-a$, respectively:

$$y(x) = \mp b \pm \sqrt{b^2 + 2ax - x^2}, \qquad 0 \leqslant x \leqslant a; \tag{2b}$$

$$y(x) = \mp b \pm \sqrt{b^2 + 2a(1-x) - (1-x)^2}, \qquad 1-a \leqslant x \leqslant 1. \tag{2c}$$

Here $b = (a^2 - h^2/4)/h$ is the distance from the circumcenter to the x axis and $R = (a^2+b^2)^{1/2} = b + h/2$.

The computational domain is bounded by a C-type far-field boundary $\Gamma = \Gamma_1 \cup \Gamma_2 \cup \Gamma_3 \cup \Gamma_4$ placed at a distance of 40 chord lengths from the airfoil (Fig. 1). Fixed values of the Mach number $M_\infty < 1$ and static temperature T_∞, as well as zero angle of attack ($\alpha = 0$), are given on the inflow part Γ_1 of Γ. On the outflow part Γ_3 of the boundary, we prescribe the static pressure p_∞ related to the temperature T and density ρ by the equation of state $p = \rho T c_v(\gamma - 1)$, where $\gamma = 1.4$. We set the free-slip condition on the upper and lower parts of the computational domain, Γ_2 and Γ_4, which are parallel to the free-stream velocity vector. The no-slip condition and

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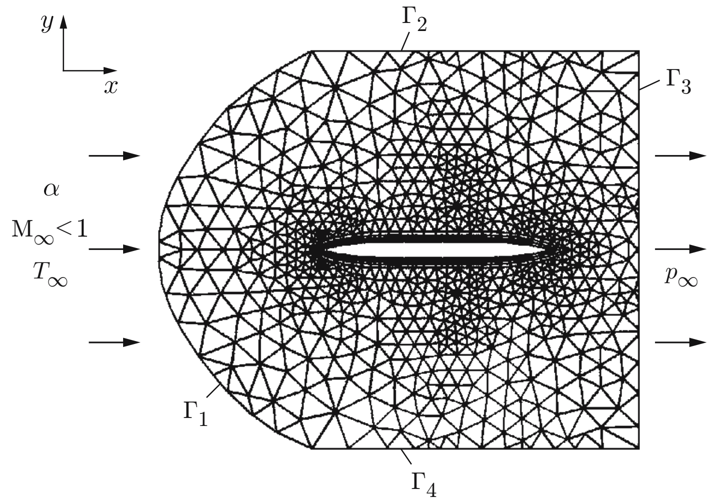

Fig. 1. Sketch of the domain and computational mesh.

the vanishing heat flux are prescribed on airfoil (2a)–(2c). The initial data are either a uniform state determined by given free-stream conditions (into which the airfoil is instantly introduced) or a non-uniform flow field obtained previously for other values of M_∞.

Solutions of the unsteady Reynolds-averaged Navier–Stokes equations were calculated with a finite-volume solver of the second-order accuracy, in which the equations were discretized in space on unstructured meshes using an upwind scheme and Van Albada limiters [11]. The time derivatives were approximated with an implicit Euler scheme. We considered a fully turbulent flow and employed the SST–k–ω turbulence model, which predicts the buffet onset quite reasonably [12, 13].

The computations were performed on hybrid meshes composed of quadrangle cells near the airfoil and triangles in the rest of the domain (see Fig. 1). To provide high-accuracy simulations of the boundary-layer flow, we chose a sufficiently small distance from the first grid point to the airfoil, so that the dimensionless distance y^+ between these grid points and the airfoil was smaller than unity [12]. The developed solver was validated by running a few benchmark problems, e.g., the one of a transonic flow past an 18% thick circular-arc airfoil. A comparison of the obtained frequencies and amplitudes of self-sustained oscillations with available experimental and numerical data for this airfoil [14, 15] demonstrated the high accuracy of the solver. For instance, at $\mathrm{Re} = 11 \cdot 10^6$, $\alpha = 0$, and gradually decreasing M_∞, the computations showed a buffet onset in the range $0.723 \leqslant M_\infty \leqslant 0.780$. A slight distinction from the test observations, which showed the buffet at $0.732 \leqslant M_\infty \leqslant 0.777$ [15], is explained by the fact that the experiments were performed in a wind tunnel with a small test section whose shape was adapted to simulate the streamlines over the airfoil in an infinite atmosphere at $M_\infty = 0.775$. At smaller values of M_∞, therefore, such a fixed adaptation simulated the streamlines inaccurately and exerted an adverse effect of the test section walls on the flow.

2. Flow Past an 8% Thick Airfoil. First, we consider airfoil (2a)–(2c) with $h = 0.08$ and $a = 0.3$. Under the free-stream conditions $\alpha = 0$, $T_\infty = 250$ K, $p_\infty = 5 \cdot 10^4$ Pa, and

$$0.850 \leqslant M_\infty \leqslant 0.877, \tag{3}$$

the computations typically exhibited the excitation of flow oscillations (due to the shock-induced boundary layer separation from the upper and lower surfaces of the airfoil in the aft region). Figure 2 demonstrates the lift coefficient as a function of time in the case of the initial data given by a uniform flow at $M_\infty = 0.857$. The Reynolds number based on the airfoil chord length of 0.5 m and $M_\infty \approx 0.864$ is $5.2 \cdot 10^6$.

To assess the numerical accuracy of the solution, we tested three meshes of 80,000, 120,000, and 180,000 cells with 30, 40, and 50 grid points across the boundary layer, respectively. A comparison of the calculated amplitudes of oscillations of the lift coefficient C_L showed that the results obtained on the second and third meshes are close enough to establish that the grid independence is virtually reached. Therefore, we employed the second mesh for further analysis of flow bifurcations and buffet.

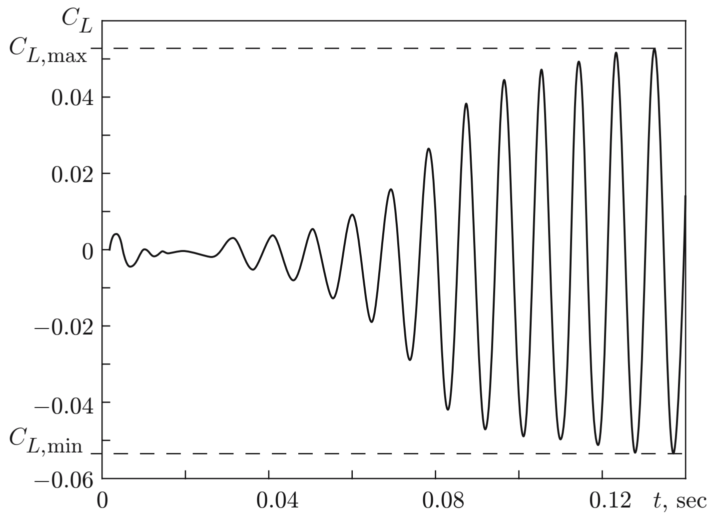

Fig. 2. Lift coefficient C_L as a function of time t for airfoil (2a)–(2c) of thickness $h = 0.08$ for the initial data determined by a uniform flow at $M_\infty = 0.857$.

Figure 3 presents the amplitudes of the lift coefficient $\Delta C_L = C_{L,\max} - C_{L,\min}$ after attaining periodic oscillations for different values of M_∞ (U_∞ is the free-stream velocity calculated via M_∞ and the sound speed determined by the temperature T_∞). The realization of a symmetric or asymmetric flow regime depends on the initial data and on the time history of the parameters M_∞ and α [6, 7].

Domains I and IV correspond to flow fields that are symmetric about the x axis after time averaging. Figures 4a and 4b present instantaneous flow fields with four supersonic zones obtained at an instant t for $M_\infty = 0.857$.

The width of domains II and III in Fig. 3 is determined by the bifurcation interval

$$0.8537 \leqslant M_\infty \leqslant 0.8612. \tag{4}$$

The corresponding asymmetric flow fields (Figs. 4c and 4d) can be obtained by an impulse perturbation of the angle of attack at $M_\infty \approx 0.8575$, then attaining periodic oscillations with a fixed amplitude, and after that gradual changing of M_∞ [7]. A mesh-refinement analysis showed that the error in calculation of the endpoints of interval (4) for a chosen turbulence model can be estimated by ± 0.0005.

For $0.8604 \leqslant M_\infty \leqslant 0.8637$, the oscillations damp out in the symmetric flow regime. This can be attributed to a shift of the separated boundary layers to a position that hinders the resonant development of perturbations. In the range

$$0.8637 < M_\infty < 0.8763, \tag{5}$$

the flow oscillations recur. At greater values of M_∞, however, the oscillations disappear again. In addition, the buffet disappears in an asymmetric flow at $M_\infty \approx 0.854$, i.e., in the beginning of the bifurcation interval (4).

In interval (5), flow oscillations are obtained under a uniform state determined by the free-stream conditions (M_∞) for the initial data. If the initial data are specified by a nonuniform flow obtained previously for different values of M_∞, then the endpoints of interval (5) change insignificantly. Therefore, one only observes a weak hysteresis in M_∞ in the case at hand.

It can be seen from Fig. 3 that there is a singular free-stream Mach number $M_\infty \approx 0.86$, at which a symmetric flow (with respect to the x axis) cannot exist, because it is impossible to realize continuous coalescence of two local supersonic zones on the airfoil surfaces with a gradual increase in M_∞ [7]. In turn, the impossibility of continuous coalescence of local supersonic zones in an inviscid flow is explained by the non-existence of an intermediate steady flow with two local supersonic zones that would have a common point on the airfoil [8, 9].

The frequency of self-sustained oscillations of an asymmetric flow in interval (4) is approximately 100 Hz. In regimes of a symmetric flow, the frequency increases from 110 to 140 Hz with M_∞ increasing in interval (3).

The computations of an inviscid flow (based on the Euler equations) past airfoil (2a)–(2c) with $h = 0.08$ showed that the bifurcation interval (4) shrinks to $0.8546 \leqslant M_\infty \leqslant 0.8560$. The values of C_L calculated in

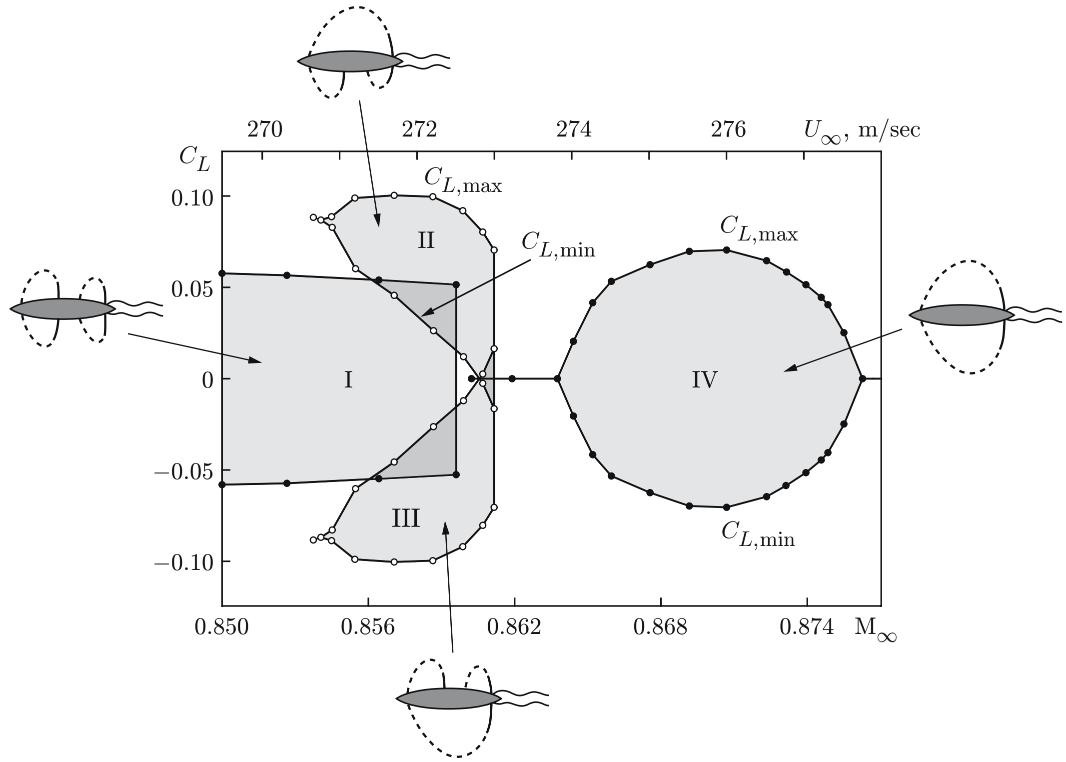

Fig. 3. Maximums and minimums of the lift coefficient C_L for an oscillatory flow past airfoil (2a)–(2c) of thickness $h = 0.08$ at $\alpha = 0$ and $\mathrm{Re} = 5.2 \cdot 10^6$: domains corresponding to flow regimes with different numbers of local supersonic zones are indicated by I–IV (four zones in domain I, three zones in domains II and III, and two zones in domain IV).

asymmetric regimes of an inviscid flow differ only by 10–25% from the average of $C_{L,\max}$ and $C_{L,\min}$ obtained for an oscillating turbulent flow. We notice that, for airfoil (1), the neglect of turbulent viscosity resulted, on the contrary, in an expansion of the bifurcation interval [10].

3. Flow Past a 7% Thick Airfoil. Now we turn to airfoil (2a)–(2c) with $h = 0.07$. Other parameters are the same as those used for the 8% airfoil: $a = 0.3$, $\alpha = 0$, $T_\infty = 250$ K, and $p_\infty = 5 \cdot 10^4$ Pa.

Reduction of the airfoil thickness entails a shift of the transonic regime to greater values of M_∞ and essentially stabilizes the flow. As is seen from Fig. 5, the oscillations no longer exist in an asymmetric flow for a 7% airfoil. At the same time, at

$$0.8762 \leqslant M_\infty \leqslant 0.8854, \tag{6}$$

there are still considerable oscillations of a symmetric flow with two supersonic zones.

The bifurcation interval $0.8658 \leqslant M_\infty \leqslant 0.8722$ in this case is shorter than (4). The singular Mach number is $M_\infty \approx 0.8706$. The neglect of turbulent viscosity entails a shift of the bifurcation interval to smaller values of M_∞ (see the dashed curves in Fig. 5).

The computations of a turbulent flow past airfoil (2a)–(2c) with $a = 0.4$, i.e., with a reduced length of the flat part and an increased length of the circular-arc portions, demonstrated a decrease in the bifurcation intervals in both cases, $h = 0.08$ and $h = 0.07$.

Conclusions. The airfoil under discussion (2a)–(2c) differs considerably from airfoil (1) due to its sharp nose with a leading-edge angle of 15°. In addition, the trailing-edge angle is also 15° instead of 0° for airfoil (1). Finally, the curvature of airfoil (2) is discontinuous at $x = a$ and $x = 1 - a$, in contrast to that of airfoil (1). Nevertheless, the diagram $C_L(M_\infty)$ depicted in Fig. 3 for airfoil (2a)–(2c) with $h = 0.08$ is similar to that for airfoil (1) [10]. This shows that the existence of bifurcations depends mainly on the length of the airfoil midportion with zero or small curvature rather than on the shape of the bow or aft parts.

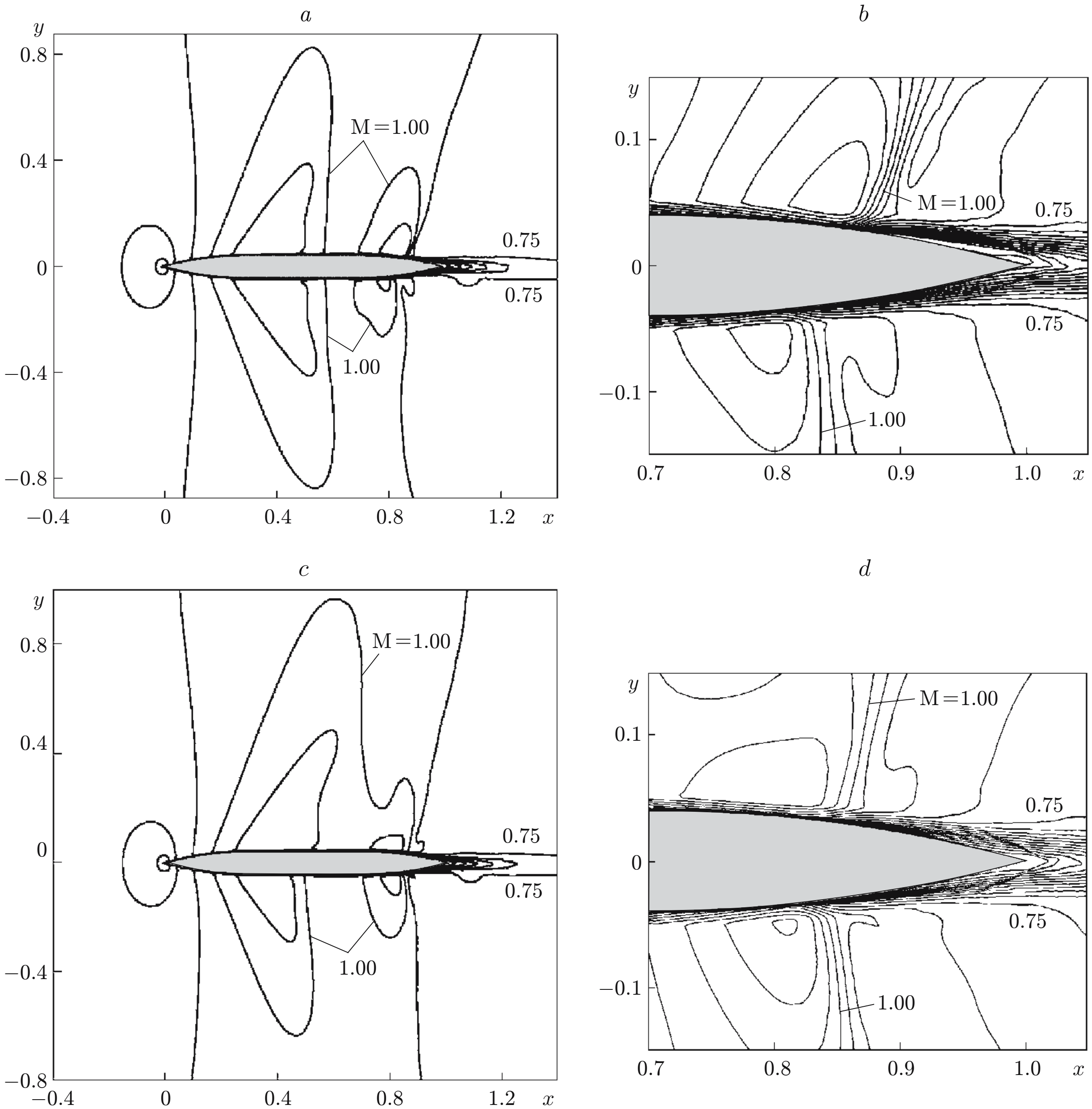

Fig. 4. Mach number contours in a symmetric (a and b) and an asymmetric (c and d) flow past airfoil (2a)–(2c) for $h = 0.08$, $\alpha = 0$, and $M_\infty = 0.857$ at an instant t: (a) and (c) positions of local supersonic zones; (b) and (d) fragments of the flow field in the aft region.

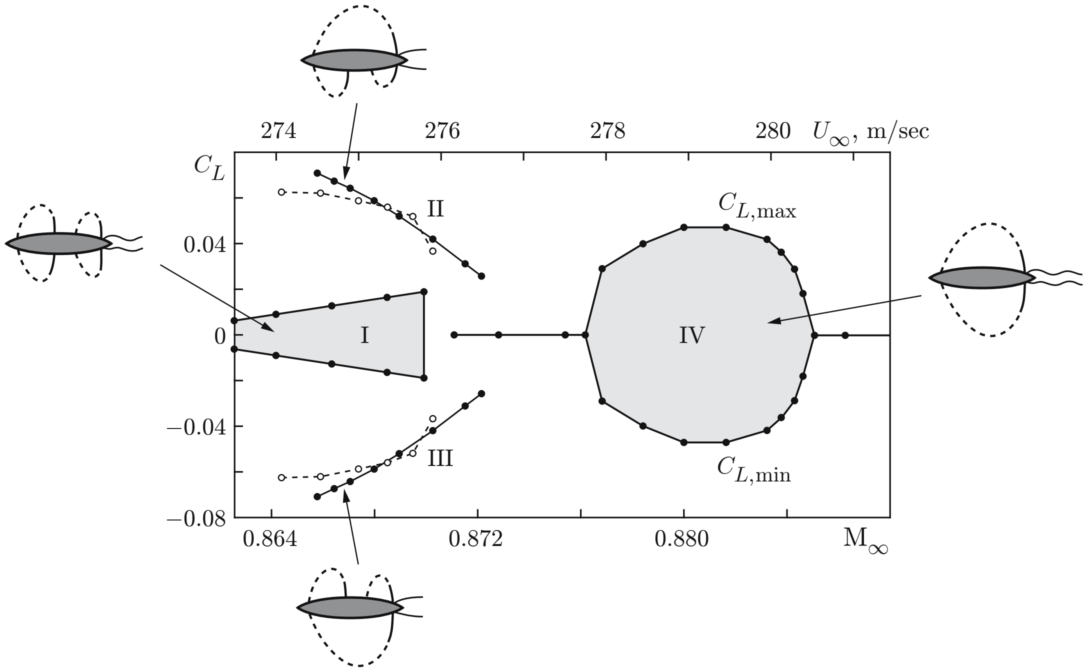

Fig. 5. Maximums and minimums of the lift coefficient C_L for airfoil (2a)–(2c) with $h = 0.07$, $\alpha = 0$, and $\mathrm{Re} = 5.3 \cdot 10^6$: the dashed and solid curves are the values calculated for asymmetric inviscid and turbulent flows, respectively; domains I–IV are the flow regimes with four supersonic zones (I), three supersonic zones (II and III), and two supersonic zones (IV).

The amplitude of oscillations of the lift coefficient C_L depends essentially on the airfoil thickness. If the thickness is reduced from 0.08 to 0.07, then the oscillations damp out in the regimes of an asymmetric flow and considerably weaken in the regimes of a symmetric flow with four supersonic zones.

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